THE EXACT SOLUTION OF FRACTIONAL ORDER INTEGRO-DIFFERENTIAL EQUATIONS USING COLLOCATION METHOD

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ABSTRACT

In this paper, the application of standard collocation method on fractional integro-differential equation was carried out by assuming a modified trial solution with Chebyshev polynomial basis. Equally spaced interior collocation points were adopted. Maple 18 was used for the computation of the four illustrative examples, for the simple demonstration of the applicability, validity and reliability of the method. It is however concluded that the method is considered as one of the novel solver of the class of fractional integro-differential equation.

1. INTRODUCTION

In recent years, a growing consideration interest in the fractional integro-differential equation is simulated, due to their numerous applications in the areas of physics, chemistry, engineering mechanics, astronomy, biology, economics and electro statistics. Differential equation involving derivatives of non-integer order have shown to be adequate models for various physical phenomena in the area like rheology, damping laws, diffusion processes. This is more realistic and it is one reason why fractional calculus has become more and more popular (see Mittal and Nigam (2008)).

In recent time, a good number of researchers have proposed and applied some efficient approximation and analytical techniques for the solution of problems of fractional calculus. Such techniques have been applied to fractional order differential equations, fractional order integral equations, and in some cases fractional order integro-differential equations. There have been attempt to solve multi-order fractional order differential equations but a complete analysis has so far not been given (Taiwo and Odetunde (2013)).

Fractional differential equations have been investigated by many researchers. Rawashdeh (2005) used the spline collocation method to approximate the solution of the fractional equation. Adomian (2013) solved multi order fractional differential equations by an iterative decomposition method. Momani (2000) obtained local and global existence and uniqueness solution of the integro-differential equation. Adomian

Decomposition method (ADM) is widely used by many researchers to solve the class of problems above in applied sciences (see Adomian (1994), Adomain (1989), Kaya and El-Sayed (2003)). Adomain (1989), provides an analytical approximation to linear and non-linear problems in this category. In Adomian method the solution is considered as the sum of an infinite series, rapidly converging to an accurate solution.

In this paper, the tradition collocation method is revisited with a little modification of the trial solution to solve fractional order integro-differential equation of the form:

$$D_*^{\alpha} y(t) = a(t)y(t) + f(t) + \int_0^t k(t,s) f\big(y(s)\big) ds, \; t \in [0\ 1] \qquad (1.1)$$

together with the initial condition

$$\sum_{i=0}^{n} a_i y^{(i)}(t_j) = b \quad j = 0,1,2,\ldots \qquad (1.2)$$

Is considered and solved numerically with the proposed method. Here D_*^{α} is the Caputo's fractional derivative and α is a parameter describing the order of the fractional derivative, and $f(y(x))$ is generally a nonlinear continuous function. Such kinds of equations arise in the mathematical modeling of various physical phenomena, such as heat conduction in materials with memory. Moreover, these equations are encountered in combined conduction, convection and radiation problems (see for example Caputo (1967), Ohnstead and Handelsman (1976), Mainardi (1997)).

2. PRELIMINARIES

In this section, we give some basic definitions and properties of relevant terms which are useful in this work.

Definition 1: An integral equation is an equation in which the unknown function y(x) appears under an integral sign. A standard integral equation is of the form

$$y(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t) y(t)\, dt \qquad (2.1)$$

Where $g(x)$ and $h(x)$ are the lower and upper limits of integration, $\boldsymbol{\lambda}$ is a constant parameter, and $K(x,t)$ is a function of two variables x and t and is called the kernel or the nucleus of the integral equation, and $f(t)$ is a smooth continuous function.

Definition 2: An integro-differential equation is an equation which involves both integral and derivatives of an unknown function.

A standard integro-differential equation is of the form:

$$y^{(n)}(x) = f(x) + \lambda \int_{g(x)}^{h(x)} K(x,t) y(t)\, dt \qquad (2.2)$$

Where $g(x), h(x), f(x)$ and the kernel $K(x,t)$ are defined in definition 1 above and n is the order of the IDE.

Equation (2.2) is called Fredholm Integro - differential Equation (FIDE) if both the lower and upper limits of the region of the integration are fixed numbers while it is called Volterra Integro-differential Equation (VIDE) if the lower limit of the region of integration is a fixed number and the upper limit is not.

An example of Fredholm Integro-differential Equation and Volterra Integro-differential Equation are given in (2.3) and (2.4).

$$y^{(n)}(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt \qquad (2.3)$$

Where a and b are fixed numbers.

$$y^{(n)}(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt \qquad (2.4)$$

where a is fixed number and $h(t)$ is a function of x.

Definition 3: A collocation method is the method of evaluating a given differential equation or integro-differential or fractional order equation at some equally spaced interior points in order to determine the valves of the unknown constants resulted from the assumed approximate solution used.

Definition 4: A real $f(t), t > 0$, is said to be the space, C_μ, $\mu \in \mathbb{R}$, if there exists a real number $p > \mu$, such that $f(t) = t^p h_1(t)$, where $h_1(t) \in C(0,\infty)$, and it is said to be in space C_μ^n if and only if $f^{(n)} \in C_\mu\ n \in N$.

Definition 5: The Reimann - Liouville fractional integral operator of order $\alpha > 0$, of a function $f \in C_\mu, \mu \geq -1$, is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)ds,\ \alpha > 0 \qquad (2.5)$$

$$J^0 f(t) = f(t) \qquad (2.6)$$

Some properties of the operator J^α, are as follows:

For $f \in C_\mu, \mu \geq -1, \alpha, \beta \geq 0$ and $\gamma \geq -1$

$$J^\alpha J^\beta f(t) = J^{\alpha+\beta} f(t), \qquad (2.7)$$

$$J^\alpha J^\beta f(t) = J^\beta J^\alpha f(t) \qquad (2.8)$$

$$J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+\alpha)} t^{\gamma+\alpha} \qquad (2.9)$$

$$D^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)} t^{\gamma-\alpha} \qquad (2.10)$$

Definition 6: The fractional derivative D^{α} of $f(t)$ in the Caputo's sense is defined as

$$D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d(\tau) \quad (2.11)$$

For $n-1 < \alpha \leq n, \quad n \epsilon N, \quad t > 0 \quad f \epsilon C_{-1}^{n}$

Definition 7: Chebyshev polynomials are sequence of orthogonal polynomials which are related to De Moivre formula and are easily defined re-cursively like Fibonacci or Lucas numbers. The Chebyshev Polynomials of degree n denoted by $T_n(x)$ of the first kind and valid in the interval $-1 \leq x \leq 1$ is defined as

$$T_n(x) = \cos(n\cos^{-1}x); \quad n > 0 \quad (2.12)$$

For $n = 0$ and 1, we obtained

$$T_0(x) = 1 \text{ and } T_1(x) = x \quad (2.13)$$

And the recurrence relation is given as:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \geq 1 \quad (2.14)$$

Also, Chebyshev polynomials in the interval of $a \leq x \leq b$ is defined as

$$T_n(x) = cos\left[n\cos^{-1}\left(\frac{2x-a-b}{b-a}\right)\right], \quad a \leq x \leq b \quad (2.15)$$

And the recurrence relation is

$$T_{n+1}(x) = 2\left(\frac{2x-a-b}{b-a}\right)T_n(x) - T_{n-1}(x), \quad n \geq 0, \quad a \leq x \leq b \quad (2.16)$$

Few terms of the Shifted Chebyshev Polynomial valid in the interval [0, 1]

Are given below

$$T_0(x) = 1$$

$$T_1(x) = 2x - 1$$

$$T_2(x) = 8x^2 - 8x + 1$$

$$T_3(x) = 32x^3 - 48x^2 + 18x - 1$$

$$T_4(x) = 128x^4 - 256x^3 + 160x^2 - 32x + 1$$

$$T_5(x) = 512x^5 - 1280x^4 + 1120x^3 - 400x^2 + 50x - 1$$

$T_6(x) = 2048x^6 - 6144x^5 + 6912x^4 - 3584x^3 + 840x^2 - 72x + 1$

$T_7(x) =8192x^7 - 2867x^6 + 39424x^5 - 26990x^4 + 9408x^3 - 1568x^2 + 98x - 1$

$T_8(x) = 32765x^8 - 131072x^7 + 212992x^6 - 40224x^5 + 84480x^4 - 21504x^3$

$+2688x^2 - 128x + 1$ (2.17)

3. METHODOLOGY OF CHEBYSHEV POLYNOMIAL AS BASIC FUNCTION ON THE GENERAL FRACTIONAL ORDER INTEGRO-DIFFERENTIAL EQUATION.

In this section, we describe our form of approximation for solving the general class of fractional order integro-differential equation considered.

Here, we assumed an approximate solution of the form:

$$y(t) \cong y_n = \sum_{j=0}^{n} a_j T_j(t) \quad (3.1)$$

Where n is the degree of the approximate used, $a_j (j \geq 0)$ are unknown constants to be determined and $T_j(j \geq 0)$, are Chebyshev polynomial defined earlier.

For the purpose of our discussion, we consider the fractional order integro-differential of the form:

$$D^{\alpha} y(t) = a(t)y(t) + f(t) + \int_0^t k(t,s) f\big(y(s)\big) ds \quad (3.2)$$

with the initial condition

$$y(0) = 0 \quad (3.3)$$

Plugging (3.3) on (3.1), we obtain

$$y(0) \cong y_n(0) = \sum_{j=0}^{n} a_j T_j(0) = 0 \quad (3.4)$$

Simplification of (3.4) gives

$$y_n(0) = \sum_{j=0}^{n} (-1)^j a_j = 0 \quad (3.5)$$

Thus, from (3.5)

$$a_0 = \sum_{j=1}^{n} (-1)^{j+1} a_j \quad (3.6)$$

Substituting (3.6) into (3.1) and after simplification we have

$$y(t) \cong \; y_n(t) = \sum_{j=1}^{n} a_j T_j^*(t) \quad (3.7)$$

Substituting (3.7) into (3.2) and after simplification we obtain

$$D^{\alpha}y_n(t) = a(t)\sum_{j=1}^{n} a_j T_j^*(t) + f(t) + \int_0^t k(t,s) f\left(\sum_{j=1}^{n} a_j T_j^*(s)\right) ds \quad (3.8)$$

Simplification of (3.8) and collecting like terms of $a_i\,(i \geq 0)$, gives

$$\left\{D^{\alpha}T_1^*(t) - p(t)T_1^*(t) - \int_0^t K(t,s)T_1^*(s)ds\right\}a_1 + \left\{D^{\alpha}T_2^*(t) - p(t)T_2^*(t) - \int_0^t K(t,s)T_2^*(s)ds\right\}a_2 + \ldots$$
$$+\left\{D^{\alpha}T_n^*(t) - p(t)T_n^*(t) - \int_0^t K(t,s)T_n^*(s)ds\right\}a_n = f(t) \quad (3.9)$$

Thus, (3.9) is then collocated at $t = t_k$ to get

$$\left\{D^{\alpha}T_1^*(t_k) - p(t_k)T_1^*(t_k) - \int_0^{t_k} K(t_k,s)T_1^*(s)ds\right\}a_1 +$$
$$\left\{D^{\alpha}T_2^*(t_k) - p(t_k)T_2^*(t_k) - \int_0^{t_k} K(t_k,s)T_2^*(s)ds\right\}a_2 + \ldots$$
$$+\left\{D^{\alpha}T_n^*(t_k) - p(t_k)T_n^*(t_k) - \int_0^{t_k} K(t_k,s)T_n^*(s)ds\right\}a_n = f(t_k) \quad (3.10)$$

where

$$t_k = a + \frac{(b-a)k}{n+1}, \quad k = 1,2,3,\ldots,n$$

The collocated (3.10) resulted into a system of equations which are then put in matrix form as

$$Ax = b$$

where

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1n} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2n} \\ & \vdots & & \ddots & \vdots \\ A_{n1} & A_{n2} & A_{n3} & \cdots & A_{nn} \end{pmatrix}$$

$$x = (x_1, x_2, \ldots, x_n)^T$$

$$b = (b_1, b_2, \ldots, b_n)^T$$

and

$$A_{11} = D^{\alpha}T_1^*(t_1)\ldots p(t_1)T_1^*(t_1) - \int_0^{t_1} K(t_1,s)T_1^*(s)ds$$

$$A_{12} = D^{\alpha}T_2^*(t_1)\ldots p(t_1)T_2^*(t_1) - \int_0^{t_1} K(t_1,s)T_2^*(s)ds$$

$$A_{13} = D^{\alpha}T_3^*(t_1)\ldots p(t_1)T_3^*(t_1) - \int_0^{t_1} K(t_1,s)T_3^*(s)ds$$

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$$A_{1n} = D^{\alpha}T_n^*(t_1)\ldots p(t_1)T_n^*(t_1) - \int_0^{t_1} K(t_1,s)T_n^*(s)ds$$

$$A_{21} = D^{\alpha}T_1^*(t_2)\ldots p(t_2)T_1^*(t_2) - \int_0^{t_2} K(t_2,s)T_1^*(s)ds$$

$$A_{22} = D^{\alpha}T_2^*(t_2)\ldots p(t_2)T_2^*(t_2) - \int_0^{t_2} K(t_2,s)T_2^*(s)ds$$

$$A_{23} = D^{\alpha}T_3^*(t_2)\ldots p(t_2)T_3^*(t_2) - \int_0^{t_2} K(t_2,s)T_3^*(s)ds$$

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$$A_{2n} = D^{\alpha}T_n^*(t_2)\ldots p(t_2)T_n^*(t_2) - \int_0^{t_2} K(t_2,s)T_n^*(s)ds$$

$$A_{31} = D^{\alpha}T_1^*(t_3)\ldots p(t_3)T_1^*(t_3) - \int_0^{t_3} K(t_3,s)T_1^*(s)ds$$

$$A_{32} = D^{\alpha}T_2^*(t_3)\ldots p(t_3)T_2^*(t_3) - \int_0^{t_3} K(t_3,s)T_2^*(s)ds$$

$$A_{33} = D^{\alpha}T_3^*(t_3)\ldots p(t_3)T_3^*(t_3) - \int_0^{t_3} K(t_3,s)T_3^*(s)ds$$

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$$A_{3n} = D^{\alpha}T_n^*(t_3)\ldots p(t_3)T_n^*(t_3) - \int_0^{t_3} K(t_3,s)T_n^*(s)ds$$

$$A_{n1} = D^{\alpha}T_1^*(t_n)\ldots p(t_n)T_1^*(t_n) \int_0^{t_n} K(t_n,s)T_1^*(s)ds$$

$$A_{n2} = D^{\alpha}T_2^*(t_n)\ldots p(t_n)T_2^*(t_n) \int_0^{t_n} K(t_n,s)T_2^*(s)ds$$

$$A_{n3} = D^{\alpha}T_3^*(t_n)\ldots p(t_n)T_3^*(t_n) \int_0^{t_n} K(t_n,s)T_3^*(s)ds$$

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$$A_{nn} = D^{\alpha}T_n^*(t_n)\dots p(t_n)T_n^*(t_n)\int_0^{t_n} K(t_n,s)T_n^*(s)\,ds \qquad (3.11)$$

After the evaluation of the integrals that appeared in the matrix, the resulting system is the solved by Mapple 18 to obtain the unknown constants that appeared in the approximate solution.

4. NUMERICAL EXAMPLES

4.1. Example 1:

Consider the following fractional order integro-differential equation, for $t \in I = [0,1]$

$$D^{0.5}y(t) = y(t) + \frac{8}{3\Gamma(0.5)}t^{1.5} - t^2 - \frac{t^3}{3} + \int_0^t y(s)\,ds \qquad (4.1.1)$$

$$y(0) = 0 \qquad (4.1.2)$$

The exact solution is $y(t) = t^2$

[see Fadi Awawdeh, 2011]

Description of Solution

Here, we assume an approximate solution of the form:

$$y_n(t) = \sum_{j=0}^{n} a_j T_j(t) \qquad (4.1.3)$$

where a_j and $T_j(t)$ are defined above

Here, we let $n = 3$

Thus (4.1.3) leads to

$$y_3(t) = a_0T_0(t) + a_1T_1(t) + a_2T_2(t) + a_3T_3(t) \qquad (4.1.4)$$

Substitution of $T_0(t), T_1(t), T_2(t), T_3(t)$ from (2.14) into (4.1.4) leads to

$$y_3 = a_0 + a_1(2t-1) + a_2(8t^2 - 8t + 1) + a_3(32t^3 - 48t^2 + 18t - 1) \qquad (4.1.5)$$

Applying the initial condition given (4.1.2) on (4.1.5), leads to

$$y_3(0) \equiv a_0 - a_1 + a_2 - a_3 = 0 \qquad (4.1.6)$$

Hence, making a_0 the subject of formula in (4.1.6) leads to

$$a_0 = a_1 - a_2 + a_3 \qquad (4.1.7)$$

Substituting (4.1.7) into (4.1.5) to obtain

$$y_3(t) = 2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3 \quad (4.1.8)$$

Thus, (4.1.8) is substituted into (4.1.1) to obtain

$$D^{0.5}\{2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3\} - \{2ta_1 + (8t^2 - 8t)a_2 + (32t^3 - 48t^2 + 18t)a_3 - \int_0^t \{2sa_1 + (8s^2 - 8s)a_2 + (32s^3 - 48s^2 + 18s)a_3\} ds = \frac{8}{3\Gamma(0.5)} t^{1.5} - t^2 - \frac{t^3}{3}$$

(4.1.9)

Hence, (4.1.9) is re-arranged to get

$$\{2D^{0.5}t - 2t - \int_0^t 2sds\}a_1 + \{D^{0.5}(8t^2 - 8t) - (8t^2 - 8t) - \int_0^t (8s^2 - 8s)ds\}a_2 + D^{0.5}(32t^3 - 48t^2 + 18t) - (32t^3 - 48t^2 + 18t) - \int_0^t (32t^3 - 48t^2 + 18t)ds\}a_3 = \frac{8}{3\Gamma(0.5)} t^{1.5} - t^2 - \frac{t^3}{3} \quad (4.1.10)$$

Thus, (4.1.10) is simplified term by term to get1

$$\left(\frac{2t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 2t - t^2\right)a_1 + \left(\frac{16t^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} - \frac{8t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 8t^2 + 8t - 8\frac{t^3}{3} + 4t^2\right)a_2 + \left(\frac{192t^{\frac{5}{2}}}{\Gamma\left(\frac{7}{2}\right)} - \frac{288t^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} + \frac{18t^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 32t^3 + 48t^2 - 18t^4 + 16t^3 - 9t^2\right)a_3 = \frac{8t^{1.5}}{3\Gamma(0.5)} - t^2 - \frac{t^3}{3} \quad (4.1.11)$$

Therefore (4.1.11) is collocated at point $t = t_k$ to get

$$\left(\frac{2t_k^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 2t_k - t_k^2\right)a_1 + \left(\frac{16t_k^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} - \frac{8t_k^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 8t_k^2 + 8t_k - 8\frac{t_k^3}{3} + 4t_k^2\right)a_2 + \left(\frac{192t_k^{\frac{5}{2}}}{\Gamma\left(\frac{7}{2}\right)} - \frac{288t_k^{\frac{3}{2}}}{\Gamma\left(\frac{5}{2}\right)} + \frac{18t_k^{\frac{1}{2}}}{\Gamma\left(\frac{3}{2}\right)} - 32t_k^3 + 48t_k^2 - 18t_k^4 + 16t_k^3 - 9t_k^2\right)a_3 = \frac{8t_k^{1.5}}{3\Gamma(0.5)} - t_k^2 - \frac{t_k^3}{3} \quad (4.1.12)$$

where,

$$t_k = \frac{k}{4}, \qquad k = 1, 2, 3$$

For $k = 1, t_1 = \frac{1}{4}$, thus substituting into (4.1.12) leads to

$$0.5658791670a_1 - 1.300677779a_2 - 17.46403084a_3 = 0.1203548612 \quad (4.1.13)$$

For $k = 2, t_1 = \frac{1}{2}$, thus substituting into (4.1.12) leads to

$$0.345769121a_1 + 0.538974505a_2 - 53.77207334a_3 = 0.2402563737 \quad (4.1.14)$$

For $k = 3, t_1 = \frac{3}{4}$, thus substituting into (4.1.12) leads to

$$-0.108089952a_1 + 2.625000000a_2 - 95.82807833a_3 = 0.2740800240 \quad (4.1.15)$$

Thus, solving (4.1.13)-(4.1.14) simultaneously using the Maple 18, we obtain

$a_1 = 0.50000$

$a_2 = 0.1250000$

$a_3 = 0.0000000$

Therefore, $a_0 = a_1 - a_2 + a_3 = 0.375000$

Substituting the values of $a_i (i = 0, 1, 2, 3)$into (4.1.8) and after simplification, the exact solution is obtained.

That is $y_3(x) = t^2$

4.2 Example 2:

Consider the following fractional order integro-differential equation

$$y^{0.75}(t) = \left(\frac{-t^2 e^t}{5}\right) y(t) + \int_0^t e^t y(s) ds + \frac{6t^{2.25}}{\Gamma(3.25)} \quad (4.2.1)$$

With the initial condition

$$y(0) = 0 \quad (4.2.2)$$

The exact solution is $y(t) = t^3$

(see Mittal R.C. and Ruchi Nigam, 2008)

Description of Solution

Following the procedure in Example 1, we obtained

$a_0 = 0.25, \quad a_1 = 0.468750, \quad a_2 = 0.187500, \quad a_3 = 0.031250$

Substituting the values of $a_i (i = 0, 1, 2, 3)$ into (4.1.8) and after simplification, the exact solution is obtained. That is

$$y_3(x) = t^3$$

4.2. Example 3:

Consider the following fraction order integro-differential equation

$$D^{0.75}y(t) - (tcost - sint)y(t) - \int_0^t tsiny(s)ds = \frac{t^{0.25}}{\Gamma(1.25)} \quad (4.3.1)$$

$$y(0) = 0 \quad (4.3.2)$$

The exact solution is $y(t) = t$

Description of solution: Following the procedure in Example 1

$a_0 = 0.5, \ a_1 = 0.5, \quad a_2 = 0, \quad a_3 = 0$

Substituting the values of $a_i(i = 0, 1, 2, 3)$ into (4.1.8) and after simplification, the exact solution is obtained. That is

$$y_3(t) = t$$

4.3. Example 4:

Consider the initial value problem that consists of the multi-fractional order integro-differential equation.

$$D^{0.5}y(t) = \frac{6}{\Gamma(3.5)}t^{2.5} - \frac{\Gamma(4)}{\Gamma(5.5)}t^{4.5} + J^{1.5}y(t) \qquad t \in [0, 1] \quad (4.4.1)$$

$$y(0) = 0 \quad (4.4.2)$$

The exact solution is $y(t) = t^3$

(see Taiwo and Odetunde, 2013)

Description of solution: Following the procedure in Example 1

$a_0 = 0.25, \ a_1 = 0.468750, \quad a_2 = 0.197500, \quad a_3 = 0.031250$

Substituting the values of $a_i(i = 0, 1, 2, 3)$ into (4.1.8) and after simplification, the exact solution is obtained. That is

$$y_3(t) = t^3$$

5. DISCUSSION AND CONCLUSION.

The traditional standard collocation method with little modification of trial solution was demonstrated on some examples of fractional integro-differential equation. The method gave an exact solution for the degree of the Chebyshev polynomial for $n \geq 3$.

We conclude therefore, that the method is very powerful and effective for solving fractional order integro differential equation.

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